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How to explain the non-zero mass of electromagnetic radiation consisting of zero-mass photons

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Abstract

The mass of electromagnetic radiation in a cavity is considered using the correct relativistic approach based on the concept of a scalar mass not dependent on the particle (system) velocity. It is shown that due to the non-additivity of mass in the special theory of relativity the ensemble of chaotically propagating mass-less photons in the cavity has a finite overall mass.

1. Introduction

It is generally accepted that the photon mass m_{ph} as the mass of an elementary particle is equal to zero [1]. Contrary to what is sometimes stated, it would have not been catastrophic and no basic laws of nature would have been violated if the quantity m_{ph} had been non-zero [2], however small enough in order not to contradict the totality of already available experimental data in many areas of physics. Nevertheless, at present electrostatic, magnetostatic and other measurements and estimations testify that $m_{\mathrm{ph}} = 0$ with a very impressive accuracy [3]. The least upper limit $m_{\mathrm{ph}}^{\mathrm{lim}} = 10^{-60}$ g was obtained by the analysis of the interstellar gas stability in the magnetic field [4].

It should be noted from the outset that the mass we are talking about is the only kind of physical mass existing in nature (in view of the equivalence between inertial and gravitational masses), which is a relativistic scalar quantity, not changing under Lorentz transformations. Therefore, the very notions of the so-called relativistic or dynamical mass (see, e.g., [5, 6]) are meaningless and contradict the above-mentioned simplest possible transformational properties of the mass m. Mass is a scalar for both an elementary particle and a macroscopic object. Unfortunately, this circumstance was fully recognized [7–13] many decades after Einstein's formulation of the special theory of relativity [14].

The erroneous notion of relativistic mass led in the past to another misconception of the finite photon mass $m_{\mathrm{ph}} \neq 0$, named a dynamical or a 'field' mass [15–18]. It seems that the methodologically incorrect link that united both notions was the implicit application of the dynamical Newton law in its original form to ordinary massive particles as well as to the photon. However, it was already shown by Planck in 1906 [19] that the dynamical law in the special theory of relativity has a different form than that in classical mechanics

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\gamma\mathbf{v}) = \mathbf{F}. \tag{1}$$

Here $\mathbf{v}$ is the velocity, $\gamma(v) = (1-v^2/c^2)^{-1/2}$, $\mathbf{F}$ is the force and m is the mass of an accelerated body. The light speed c enters the famous relativistic factor $\gamma(v)$ and is the limiting value for v. The time derivative is applied to the relativistic momentum $\mathbf{p} = m\gamma(v)\mathbf{v}$ constituting the first three components of the four-dimensional energy–momentum vector [7, 8] and differing from the Newton expression $m\mathbf{v}$.

The only mass used in equation (1) is the conventional scalar mass m, which is often called the 'rest mass' by the adepts of the relativistic mass. As we have already indicated [3, 4], the mass m is zero for the photon. Therefore, this particle cannot be accelerated or gain any mass of the mysterious 'field' (or 'dynamic') origin. This means that the original photon mass m_{ph} remains zero, whatever the photon frequency, energy or momentum.

On the other hand, one can examine some *finite* volume containing electromagnetic radiation and analyse, whether *its mass* is zero or non-zero. By no means can an immediate (and wrong!) conclusion be made that a finite amount of radiation consisting of zero-mass photons has zero mass itself. Although the failure of such a generalization seems almost evident, we have never come across any elementary proof of the finite mass of the electromagnetic field on the basis of the photon approach. This proof is presented below and applications are made to some problems which are interesting for university undergraduates. It is even more important since the usage of misconceptions mentioned above renders the correct analysis impossible, so that the whole issue should be reconsidered at greater length.

2. The non-additivity of masses in the special theory of relativity

In relativistic theory the mass is not an additive quantity [10, 11, 20], contrary to what seems so trivial for Newtonian mechanics. This very often neglected fact can be inferred from a consideration of an energy E_{12} and momentum $\mathbf{p}_{12}$ for a composite consisting of two non-interacting particles with partial energies $E_{1,2}$ and momenta $\mathbf{p}_{1,2}$. The particles were chosen as non-interacting, because in the special theory of relativity the position-dependent potential energy $U_{12}(\mathbf{r}_1, \mathbf{r}_2)$ cannot be unambiguously defined for the general case of constituents moving relative to each other. This difficulty leads to the approximate character of the corresponding Planck equation (1), because the conventional expression for the inter-particle force $\mathbf{F}_{12}(\mathbf{r}_1, \mathbf{r}_2) \equiv -\nabla U_{12}(\mathbf{r}_1, \mathbf{r}_2)$ does not take into account the retardation of the interaction due to the finiteness of c. This circumstance might not be practically important but its existence should be always kept in mind for principal reasons. In our case of *linear* electrodynamics, photons do not interact and all such reservations concerning possible forces become redundant.

The non-additivity of masses can be easily obtained from the *additivity* of conserved properties: energies and momenta for particles in question. Namely,

$$E_{12} = E_1 + E_2 \tag{2}$$

and

$$\mathbf{p}_{12} = \mathbf{p}_1 + \mathbf{p}_2. \tag{3}$$

At the same time a dispersion law of the free elementary particle in the special theory of relativity has a more sophisticated form than in Newtonian mechanics and can be readily obtained either from the four-dimensional Minkowski approach [21, 22] or Einstein's composition law for velocities [23]:

$$E = c\sqrt{\mathbf{p}^2 + m^2c^2}. \tag{4}$$

From equations (2), (3) and (4) one obtains the mass m_{12} of the composite object

$$m_{12} = \sqrt{\frac{E_{12}^2}{c^4} - \frac{|\mathbf{p}_{12}|^2}{c^2}} = \sqrt{\frac{(E_1 + E_2)^2}{c^4} - \frac{(\mathbf{p}_1 + \mathbf{p}_2)^2}{c^2}}. \tag{5}$$

One can easily ascertain that $m_{12} \neq m_1 + m_2$, contrary to what is characteristic of the non-relativistic case. Equation (5) can be generalized to systems consisting of more than two components. The very significant property of this equation is the dependence of the combined mass on the relative motion of the constituents.

3. Application to photons

The relativistic equation (5) can be used not only for massive particles but also for photons, notwithstanding the zero value of mass for the latter. Really, the only difference arising for a photon consists of the reduction of the square-root equation (4) into a linear one

$$E = c|\mathbf{p}|, \tag{6}$$

whereas the relationship (5) remains intact. Let us restrict ourselves, as before, to the bi-photon complex.

Consequently, two limiting possibilities become evident [10, 12]. Specifically, if photon momenta are parallel, the momentum sum in (5) can be rewritten as follows,

$$(\mathbf{p}_1 + \mathbf{p}_2)^2 = (p_1\mathbf{n} + p_2\mathbf{n})^2 = \mathbf{n}^2(p_1 + p_2)^2 = \left(\frac{E_1}{c} + \frac{E_2}{c}\right)^2 = \frac{1}{c^2}(E_1 + E_2)^2, \tag{7}$$

where $\mathbf{n}$ is the unit vector in the direction of the photon movement. From equations (5) and (7) it comes about that $m_{12}^{\text{par}} = 0$. The same is true for *any* photon number. It means that the parallel light beam is *mass-less*, although it has a finite momentum and can exert pressure on a target, which can be calculated either by the classical Maxwell theory [24] or using [25] the heuristic Einstein's photon picture [26]. Of course, the result is the same in both approaches.

On the other hand, if the photon momentum directions are opposite, i.e. $\mathbf{n}_1 = -\mathbf{n}_2$, the momentum sum in (5) equals zero and the mass of the bi-photon complex is finite,

$$m_{12}^{\text{antipar}} = \frac{E_{12}}{c^2} = \frac{(E_1 + E_2)}{c^2} = \frac{(p_1 + p_2)}{c} = \frac{h(\nu_1 + \nu_2)}{c^2}. \tag{8}$$

Here h is Planck's constant, ν_1 and ν_2 are the photon frequencies. This result shows, in particular, that the conventional elementary interpretation of the particle–antiparticle annihilation is wrong. Indeed, if an electron e^- with a momentum $\mathbf{p}_e = m_e\gamma\mathbf{v}_e$ and a positron e^+ with an opposite momentum $\mathbf{p}_p = -\mathbf{p}_e$ collide and annihilate in the final state one has radiation quanta instead of matter. Since conservation laws of both energy and momentum are obeyed, the resulting two photons have equal energies $E_1 = E_2 = h\nu = m_e\gamma c^2$ and momenta $\mathbf{p}_{ph1} = h\nu\mathbf{n}_1/c = -\mathbf{p}_{ph2} = -h\nu\mathbf{n}_2/c$, where $\mathbf{n}_1 = -\mathbf{n}_2$ similar to the general case considered above. The resulting finite mass of this particular bi-photon complex

$$m_{12}^{\mathrm{annih}} = \frac{E_{12}}{c^2} = \frac{(E_1 + E_2)}{c^2} = \frac{2h\nu}{c^2} = 2m_{\mathrm{e}}\gamma = \frac{E_{\mathrm{e}} + E_{\mathrm{p}}}{c^2} \tag{9}$$

is exactly equal to the initial mass $2m_{\mathrm{e}}$ of the electron–positron system times the relativistic factor γ.

The last equality in (9) was based on the correct Einstein's formula linking the system *rest* energy E_0 and its mass m [10–12, 27]

$$E_0 = mc^2. \tag{10}$$

For colliding electrons and positrons the energy E_0 must be calculated by definition in the reference frame, where the centre of inertia is at rest. Nevertheless, the kinetic energy of two particles moving 'inside' the system, which constitutes the simple system in question, must be taken into account. Therefore, $m \equiv m_{\mathrm{e+p}} = 2m_{\mathrm{e}}\gamma$, where the factor γ makes allowance for the kinetic energies of both electrons and positrons. It is remarkable that actually

$$m_{12}^{\mathrm{annih}} = m_{\mathrm{e+p}}, \tag{11}$$

although usually in textbooks the above considered annihilation process is treated as such, where the overall mass is not conserved. In general, however, the mass may or may not conserve in relativistic physics, where the Lavoisier law is not necessarily valid.

One should stress that for an arbitrary angle between two photon momenta in the bi-photon complex its mass m_{12} can have any value in the range

$$0 \leqslant m_{12} \leqslant \frac{h\,(\nu_1 + \nu_2)}{c^2}. \tag{12}$$

4. Electromagnetic radiation in a cavity

Now let us examine a vessel with a closed cavity of volume V at the thermodynamic equilibrium. At any temperature, T, some amount of electromagnetic radiation will be contained inside the cavity [24]. For our didactic purposes, it is instructive to consider this radiation as a gas of photons. Of course, strictly speaking the radiation in the cavity should be treated by quantum electrodynamics [1, 28], a beautiful theory going far beyond our elementary model considerations. Moreover, the very notion of a 'photon' does not work in many situations [28, 29]. Nevertheless, in the studied case, where coherence problems are not on the agenda, we can show that good old Einstein's heuristic idea [26] is still helpful.

Our analysis is based on the assumption of the chaotic character of the photon motion in the cavity. The assumption seems quite reasonable because there is no preferable direction of photon random flights or any preferable absorption and radiation sites. Hence, the photon contribution to the mass of the cavity can be estimated by averaging over all possible directions of photon propagation. It is worth noting that an analogous contribution to the mass of a box from the kinetic energy of confined non-interacting particles has been calculated recently [30]. Thus, the radiation is considered as a chaotic ensemble of relativistic particles with momenta $\mathbf{p}_{\mathrm{ph},i} = \mathbf{n}_i h\nu_i/c$, where the subscript '$i$' denotes the ith photon. We are no longer restricted to two photons, because the photon spectrum and numbers are determined by T. The total momentum of the radiation

$$\mathbf{P}_{\mathrm{rad}} = \sum_i \mathbf{p}_{\mathrm{ph},i} \tag{13}$$

is equal to zero due to the equiprobability of any $\mathbf{n}_i$ and $-\mathbf{n}_i$. Similarly to equation (8) this means that the total mass of the radiation M_{rad} is non-zero and is given by the relativistic expression

$$M_{\text{rad}} = \left[\left(\sum_i \frac{E_{\text{ph},i}}{c^2}\right)^2 - \left(\sum_i \frac{\mathbf{p}_{\text{ph},i}}{c}\right)^2\right]^{\frac{1}{2}} = \left[\left(\frac{E_{\text{rad}}}{c^2}\right)^2 - \left(\frac{\mathbf{P}_{\text{rad}}}{c}\right)^2\right]^{\frac{1}{2}}$$
$$= \left[\left(\frac{E_{\text{rad}}}{c^2}\right)^2\right]^{\frac{1}{2}} = \frac{E_{\text{rad}}}{c^2}\,, \tag{14}$$

where E_{rad} is the radiation energy.

Result (14) is quite natural and fully agrees with Einstein's fundamental relationship (10). The existence of the mass component (14) testifies that the total mass of the cavity includes not only the mass of the enclosed gaseous particles of whatever origin but that of the confined radiation as well. The gas particle and photon mass contributions are caused both by particle masses and the *dynamical* terms: the gas internal energy and the electromagnetic energy. Due to the absence of the mass additivity in the special theory of relativity, the zero value of the photon mass is not an obstacle to the appearance of the total radiation mass $M_{\text{rad}} \neq 0$. This circumstance has been overlooked so far.

We stress that the very effect of the mass non-additivity has a dynamical nature, i.e. it results from processes in the intrinsic frame of reference. In contrast, if one considers the same system from the viewpoint of another reference frame, relative to which the hollow vessel moves as a whole with a certain velocity v_{ves}, this *kinematical* circumstance *does not change* the vessel mass, although the total energy E of the vessel will change according to the relativistic law

$$E = E_0 \times \gamma(v_{\text{ves}}). \tag{15}$$

Here E_0 is given by Einstein's formula (10) with *the frame-independent scalar mass* m_{ves} of the vessel.

For the specific estimations of the dynamical mass terms defined above we take into account the T uniformity in the equilibrium state for all the cavity contents and the vessel walls. First let us examine the contribution $M_{\text{gas}}^{\text{dynam}}$ of the internal energy of a gas which is chaotically moving in the cavity. For simplicity, the gas is assumed to be monatomic and ideal. The latter assumption is reasonable for a room temperature 300 K and an ambient pressure P of one atmosphere. These conditions will be hereafter assumed to be valid. (We shall not estimate a trivial although substantially larger 'non-relativistic' contribution $M_{\text{gas}} = m_{\text{at}} \times nV$, where m_{at} is the atomic mass and n is the number of atoms in the unit volume.)

The internal energy $E_{\text{gas}}^{\text{dynam}}$, which for an ideal gas coincides with the kinetic energy of its molecules, is given by the equation (see, e.g., [31])

$$E_{\text{gas}}^{\text{dynam}} = \tfrac{3}{2} n k_{\text{B}} T V. \tag{16}$$

Here k_{B} is the Boltzmann constant. Once more using the basic equation (10) we obtain

$$M_{\text{gas}}^{\text{dynam}} = \frac{E_{\text{gas}}^{\text{dynam}}}{c^2}. \tag{17}$$

For the adopted standard conditions and $V = 1\ \text{cm}^3$ we estimate $M_{\text{gas}}^{\text{dynam}} \approx 1.6 \times 10^{-15}$ g.

Bearing in mind equation (14) the next step is to calculate E_{rad}. This energy equals uV, where u is the total bulk density of radiation. In our case it is a black-body equilibrium radiation given by the Stefan–Boltzmann–Planck law [24, 31]

$$u = \frac{8\pi k_{\text{B}}^4 T^4}{15 c^3 h^3}. \tag{18}$$

Since the notion of the mass of electromagnetic radiation has been used, it should be indicated that in the analysis of mutual transformations among elementary particles it is enough to deal with energies and momenta for those particles, including radiation quanta. The additional introduction of the radiation mass as its energy divided by c^2 has no special significance being a trivial consequence of the energy conservation law [32]. However, when one considers an overall mass of a certain body, e.g., the hollow vessel discussed here, to make no allowance for the electromagnetic radiation mass inside the cavity will lead to fundamental error, notwithstanding the numerical smallness of the term in question.

If we know all the constants in equation (18), we estimate M_{rad} as 7×10^{-28} g, i.e. it is less than the electron mass. Nevertheless, one should keep in mind that M_{rad} rapidly grows with T, so that, e.g., for 1200 K the radiation mass in the cavity becomes equal to 1.8×10^{-25} g. It is no wonder that $M_{\mathrm{rad}} \ll M_{\mathrm{gas}}^{\mathrm{dynam}}$ for normal ambient conditions, when matter dominates over radiation in the equilibrium state. Nevertheless, both tiny corrections M_{rad} and $M_{\mathrm{gas}}^{\mathrm{dynam}}$ to the vessel mass should be taken into account from the principal point of view.

5. Conclusions

Our study has shown an example of how the abandonment of outdated spurious notions, such as the 'relativistic mass' and finite 'photon mass', can lead to useful principal results. In particular, the consistent relativistic approach adopted here enabled us to resolve the imaginary conflict between the finite mass of the electromagnetic field contained in a closed volume and the zero mass of photons as its elementary constituents. In everyday life finite radiation mass can be neglected for all practical purposes. Nevertheless, it is important to introduce this finiteness for students as an example of the universal applicability of Einstein's famous relationship between mass and energy in its correct original form [27]. The analysis carried out above has also demonstrated the productivity of the photon concept [26] despite its apparently non-rigorous character [28, 29].

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